4. Locus

The path traced by a moving point under certain geometrical condition(s) is called the locus of the moving point.

An equation is said to be the equation of locus of a moving point if :

- the coordinates of every point on the locus satisfy the equation
- the coordinates of any point satisfy the equation, then the point must lie on the locus

Working Rule For Finding Equation of the Locus of a Point :

1. If x and y coordinates of the moving points are expressed in term of a parameter, say t, then eliminate t to obtain the relation in x and y. Simplifying this relation will give the required locus.
2. If some geometrical condition(s) is(are) given, then:

- Take the coordinates of the variable points as (h, k).
- Express the given geometrical condition in terms of h and k.
- Eliminate the arbitrary variables to obtain the relation in terms of h and k, i.e. the relation must only involve h, k and known quantities.
- When we shift the origin to a point or rotate the axes, the coordinates of the given point change with respect to that point or axes.
- Let $f(x, y) = 0$ be the equation of a curve with respect to the origin O (0, 0). When the origin is shifted to the point (a, b), then the equation of the curve with respect to the new coordinate system will be $f(x + a, y + b) = 0$.
- When the coordinate axes are rotated through an angle of θ about the origin, then the coordinates of point P are shifted to (X, Y) in the new system. The relation between the old and the new coordinates of P is given as $x = X\cos\theta - Y\sin\theta$ and $y = X\sin\theta + Y\cos\theta$
- Let us consider a curve $f(x, y) = 0$ in a two dimensional system. When the system is rotated about the origin by an angle θ, then the equation of the curve, with respect to the new coordinate system, will be $f(x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta) = 0$.
- If axes are rotated through an angle of $(-\theta)$, then the relation between the old and the new coordinates of the point P is given as $x = X\cos\theta + Y\sin\theta$ and $y = -X\sin\theta + Y\cos\theta$

Note: If the origin is shifted to the point (a, b) and axes are rotated through an angle of θ, then the equation of the curve $f(x, y) = 0$ becomes $f(x\cos\theta - y\sin\theta + a, x\sin\theta + y\cos\theta + b) = 0$.